Quantum electromdynamics in a linear absorbing dielectric medium

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Abstract

The eletromagnetic field in a linear absorptive dielectric medium, is quantized in the framework of the damped polarization model. A Hamiltonian containing a reservoir with continuous degrees of freedom, is proposed. The reservoir minimally interacts with the dielectric polarization and the electromagnetic field. The Lagevin-Schrodinger equation is obtained as the equation of motion of the polarization field. The radiation reaction electromagnetic field is considered. For a homogeneous medium, the equations of motion are solved using the Laplace transformation method.

1 Introduction

In a homogeneous and nondispersive medium, the photon is associated with only the transverse cmponents of the electromagnetic field, in contrast in an inhomogeneous nondispersive medium, the transverse and the longitudinal degrees of freedom are coupled. In this case the quantization of the electromagnetic field can be accomplished by employing a generalized gauge [1,2], that is, $\vec{\nabla}.(\varepsilon(\vec{r})\vec{A}) = 0$, where $\varepsilon(\vec{r})$ is the space dependent dielectric function.

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Generalization of this quantization to an anisotropic nondispersive medium is straightforward by using of the gauge $\sum_{j=1}^{3}\frac{\partial}{\partial x_j}(\varepsilon_{ij}(\vec{r})\vec{A}_j) = 0$ [3].

The quantization in a dispersive and absorptive dielectric, represents one of the most and interesting problems in quantum optics, because it gives a rigorous test of our understanding of the interaction of light with matter. The dissipative nature of a medium is an immediate consequence of its disspersive character and vice versa according to the Kramers-Kronig relations. This means that the validities of the electromagnetic quantization in nondissipative but dispersive media is restricted to some range of frequencies for which the imaginary part of frequency dependent dielectric function is negligible, otherwise there will be inconsistencies.

In the scheme of Lenac [4], for dispersive and nonabsorptive dielectric media starting with the fundamental equations of motion , the medium is described by a dielectric function $\varepsilon(\vec{r},\omega)$, without any restriction on its spatial behavior. In this scheme it is assumed that there is no losses in the system, so the dielectric function is real for the whole space. The procedure is based on an expansion of the total field in terms of the coupled eigenmodes, orthogonality relations are derived and equal-time commutation relations are discussed.

Huttner and Barnett [5] have presented a canonical quantization for the electromagnetic field inside the dispersive and absorptive dieletrics based on a microscopic model in which the medium is represented by a collection of the interacting matter fields. The absorptive character of the medium is modeled through the interaction of the matter fields with a reservoir consisting of a continuum of the Klein-Gordon fields. In their model, eigen-operators for the coupled systems are calculated and the electromagnetic field has been expressed in terms of them, the dielectric function is derived and it is shown to satisfy the Kramers-Kronig relations.

Matloob [6] has quantized the electromagnetic field in a linear isotropic medium by associating a damped harmonic oscillator with each mode of the radiation field. A canonical approach has been used to quantizing a damped quantum oscillator. The conjugate momentum is defined and a quantum mechanical Hamiltonian is introduced.

Gruner and Welsh [7] have given a quantization scheme for the radiation field in dispersive and absorptive linear dielectric by starting from the phenomenological Maxwell equations, there the properties of the dielectric are described by a permitivity consistent with the Kramers-Kronig relations, an expansion of the field operators is performed based on the Green function

of the classical Maxwell equations which preserves the equal-time canonical commutation relations. In particular, in frequency intervals with approximately vanishing absorption, the concept of quantization through mode expansion for dispersive dielectrics is recognized. The theory further reveals that weak absorption gives rise to space-dependent mode operators which spatially evolve according to the quantum Langevin equations in the space domain.

Suttorp and Wubs [8] in the framework of a damped polarization model, have quantized the electromagnetic field in an absorptive medium with spatial dependendence of its parameters. They have solved equations of motion of the dielectric polarization and the eletromagnetic field by means of the Laplace transformation for positive and negative times. The operators that diagonalize the Hamiltonian are found as linear combinations of canonical variables with coefficients depending on the electric susceptibility and the dielectric Green function. The time dependence of the electromagnetic field and dielectric polarization are determined. Present authors [9] have given a model in which a linear absorptive dielectric is modeled by two independent reservoirs which are called electrical and magnetic reservoirs. In this model, the electrical and magnetic polarization densities, for the reservoirs, are defined. The electrical and magnetic polarization densities interact with the displacement vector field and the magnetic field respectively. Both structural and Maxwell equations are obtained as the Heisenberg equations of motion. There are some another approaches for quantizing the electromagnetic field, see for example [10-19].

In this paper in the framework of the damped polarization model, we introduce a method in which the electromagnetic and dielectric polarization fields, interact with a reservoir through a minimal coupling term. We obtain the Langevin-Schrodinger equation as the equation of motion of the dielectric polarization and calculate the radiation reaction electromagnetic field in terms of the polarization field. Finally we solve the equations of motion in a homogeneous dielectric using the Laplace transformation.

2 Quantum dynamics

We take a model in which the polarization of the dielectric and the electromagnetic field, as quantum fields, interact with a reservoir through a minimal coupling iterm. In this model we can obtain the Maxwell equations and the

Langevin-Schrodinger equation as the equations of motion of the polarization density of the dielectric. The vector potential for the electromagnetic field in the coulomb gauge, $\nabla.\vec{A}=0$, can be expanded in terms of the plane waves

$$\vec{A}(\vec{r},t)=\sum_{\lambda=1}^{2}\int d^3\vec{k}\sqrt{\frac{\hbar}{2(2\pi)^3\varepsilon_0\omega_{\vec{k}}}}[a_{\vec{k}\lambda}(t)e^{i\vec{k}\cdot\vec{r}}+a^{\dagger}_{\vec{k}\lambda}(t)e^{-i\vec{k}\cdot\vec{r}}]\vec{e}(\vec{k},\lambda), \qquad (1)$$

where $\omega_{\vec{k}}=c|\vec{k}|$ and ε_0 is transitivity of vacuum, $\vec{e}(\vec{k},\lambda)$ are polarization unit vectors

$$\begin{aligned}&\vec{e}(\vec{k},\lambda)\cdot\vec{e}(\vec{k},\lambda')=\delta_{\lambda\lambda'},\\ &\vec{k}\cdot\vec{e}(\vec{k},\lambda)=0, \qquad\qquad \lambda=1,2,\end{aligned} \qquad (2)$$

Canonical momentum density of the electromagnetic field is defined by

$$\vec{\pi}_F(\vec{r},t)=-i\varepsilon_0\sum_{\lambda=1}^{2}\int d^3\vec{k}\sqrt{\frac{\hbar\omega_{\vec{k}}}{2(2\pi)^3\varepsilon_0}}[a_{\vec{k}\lambda}(t)e^{i\vec{k}\cdot\vec{r}}-a^{\dagger}_{\vec{k}\lambda}(t)e^{-i\vec{k}\cdot\vec{r}}]\vec{e}(\vec{k},\lambda). \qquad (3)$$

The creation and annihilation operators $a^{\dagger}_{\vec{k}\lambda}$, $a_{\vec{k}\lambda}$, in any instant of time, satisfy the comutation relations

$$[a_{\vec{k}\lambda}(t),a^{\dagger}_{\vec{k}'\lambda'}(t)]=\delta_{\lambda\lambda'}\delta(\vec{k}-\vec{k'}), \qquad (4)$$

and their time dependence is to be determined from the total Hamiltonian. Comutation relations (4) lead to

$$[\vec{A}_i(\vec{r},t),\pi_{Fj}(\vec{r'},t)]=i\hbar\delta^{\perp}_{ij}(\vec{r}-\vec{r'}), \qquad (5)$$

where $\delta^{\perp}_{ij}(\vec{r}-\vec{r'})=\frac{1}{(2\pi)^3}\int d^3\vec{k}e^{i\vec{k}.(\vec{r}-\vec{r'})}(\delta_{ij}-\frac{k_ik_j}{|\vec{k}|^2})$, is the transverse delta function. The Hamiltonian of the quantum electromagnetic field is

$$H_F=\int d^3\vec{r}[\frac{\pi_F^2}{2\varepsilon_0}+\frac{(\nabla\times\vec{A})^2}{2\mu_0}]=\sum_{\lambda=1}^{2}\int d^3\vec{k}\hbar\omega_{\vec{k}}(a^{\dagger}_{\vec{k}\lambda}a_{\vec{k}\lambda}+\frac{1}{2}). \qquad (6)$$

The existence of electrical field inside a dielectric medium, cause electrical charges to move from their stable positions. Let $\alpha(\vec{r})$ be the density of the displaced charges and $\vec{Y}(\vec{r},t)$ be their displacement, then the polarization

density induced in the medium is $\vec{P} = \alpha\vec{Y}$. We can take $\vec{Y}$ as a quantum field and expand it in terms of the plane waves

$$\vec{Y}(\vec{r},t) = \sum_{\nu=1}^{3} \int \frac{d^3\vec{k}}{\sqrt{2(2\pi)^3}} [d_{\vec{k}\nu}(t) e^{i\vec{k}.\vec{r}} + d^{\dagger}_{\vec{k}\nu}(t) e^{-i\vec{k}.\vec{r}}] \vec{u}_\nu(\vec{k}), \tag{7}$$

where $\vec{u}_\nu(\vec{k}) = \vec{e}(\vec{k},\nu)$ for $\nu = 1,2$ and $\vec{u}_3(\vec{k}) = \hat{k} = \frac{\vec{k}}{|\vec{k}|}$. The sum over $\nu = 1,2$, gives the transverse component of $\vec{Y}$ and $\nu = 3$, is the longitudinal component. We can write the canonical momentum density of the quantum field $\vec{Y}$ as

$$\vec{\pi}_Y(\vec{r},t) = -i \sum_{\nu=1}^{3} \int d^3\vec{k} \sqrt{\frac{\hbar}{2(2\pi)^3}} [d_{\vec{k}\nu}(t) e^{i\vec{k}.\vec{r}} - d^{\dagger}_{\vec{k}\nu}(t) e^{-i\vec{k}.\vec{r}}] \vec{u}_\nu(\vec{k}). \tag{8}$$

Imposing the commutation relations

$$[d_{\vec{k}\nu}(t), d^{\dagger}_{\vec{k'}\nu'}(t)] = \delta_{\nu\nu'} \delta(\vec{k} - \vec{k'}), \tag{9}$$

on $d_{\vec{k}\nu}$ and $d^{\dagger}_{\vec{k}\nu}$, leads to the following commutation relations for vector fields $\vec{Y}$ and $\vec{\pi}_Y$

$$[\vec{Y}_i(\vec{r},t), \vec{\pi}_{Yj}(\vec{r'},t)] = i\hbar\delta_{ij}\delta(\vec{r} - \vec{r'}). \tag{10}$$

Let us assume a spring force $-\rho(\vec{r})\omega_0^2(\vec{r})\vec{Y}$, exerted on an element of the medium with volume $d^3\vec{r}$ centered in $\vec{r}$, where $\rho(\vec{r})$, is the mass density of the displaced charges, then the Hamiltonian of the quantum field $\vec{Y}$ can be written as

$$H_Y = \int d^3\vec{r} [\frac{\vec{\pi}_Y^2}{2\rho} + \frac{1}{2}\rho(\vec{r})\omega_0^2(\vec{r})\vec{Y}^2]. \tag{11}$$

If the damping forces together with the electrical force are exerted on the elements of the dielectric, then in the damped polarization model, it will be assumed that the damping forces are caused by a quantum field, which is called the reservoir or the environment. In this model, the equation of motion of the quantum field $\vec{Y}$ and the reservoir together with the Maxwell equations, can be obtained from the Heisenberg equations using the total Hamiltonian

$$H = \int d^3\vec{r} [\frac{(\vec{\pi}_Y - \alpha\vec{A} - \vec{R})^2}{2\rho} + \frac{1}{2}\rho(\vec{r})\omega_0^2(\vec{r})\vec{Y}^2 + \frac{(\alpha\vec{Y}) \cdot (\alpha\vec{Y})^{\parallel}}{2\varepsilon_0}] + H_F + H_R, \tag{12}$$

where $(\alpha\vec{Y})^{\parallel}$, is the longitudinal component of the polarization density and H_R is the Hamiltonian of the reservoir or environment defined as

$$H_R = \sum_{\nu=1}^{3} \int d^3\vec{k} \int d^3\vec{q}\hbar\omega_{\vec{k}}[b_\nu^\dagger(\vec{k},\vec{q},t)b_\nu(\vec{k},\vec{q},t) + \frac{1}{2}], \tag{13}$$

with the commutation relations

$$[b_\nu(\vec{k},\vec{q},t), b_{\nu'}^\dagger(\vec{k'},\vec{q'},t)] = \delta_{\nu\nu'}\delta(\vec{k}-\vec{k'})\delta(\vec{q}-\vec{q'}). \tag{14}$$

The operator $\vec{R}(\vec{r},t)$, plays a crucial rule in the interaction between the reservoir with the electromagnetic field and the quantum field $\vec{Y}$, by definition

$$\vec{R} = \sum_{\nu=1}^{3} \int d^3\vec{k} \int \frac{d^3\vec{q}}{\sqrt{(2\pi)^3}}[f(\omega_{\vec{k}},\vec{r})b_\nu(\vec{k},\vec{q},t)e^{i\vec{q}\cdot\vec{r}} + f^*(\omega_{\vec{k}},\vec{r})b_\nu^\dagger(\vec{k},\vec{q},t)e^{-i\vec{q}\cdot\vec{r}}]\vec{u}_\nu(\vec{q}). \tag{15}$$

The function $f(\omega_{\vec{k}},\vec{r})$, is called a coupling function which is position dependent in an inhomogeneous medium. According to the total Hamiltonian (12), the reservoir interacts with both the electromagnetic field and the quantum field $\vec{Y}$, while in previous models, the reservoir was interacting only with the field of the polarization density of the dielectric.

2.1 The equation of motion of the vector field $\vec{Y}$

Because the damping forces are exerted on the elements of the dielectric, the quantum field $\vec{Y}$, describes a dissipative quantum system. Such a system can be described in terms of the Langevin equation [20] which has a broad and general application. This description can be formulated using a coupling between the system and a quantum mechanical heat-bath. The Heisenberg equation for the dissipative system, takes the form of Langevin-Schrodinger equation. The equation of motion of the position operator $\vec{x}(t)$, is

$$m\ddot{\vec{x}} + \int_0^t dt'\mu(t-t')\dot{\vec{x}}(t') + V(\vec{x}) = \vec{F}_N(t), \tag{16}$$

where the coupling with the heat-bath is described by two terms, a term that describes the absorption of energy by the heat-bath which is characterized

by the memory function $\mu(t)$ and a fluctuating term, characterized by the operator valued noise force $\vec{F}_N(t)$. Both terms are necessary for a consistent description of a quantum damped system. In the present model, with the proposed Hamiltonian (12), we can obtain the Langevin-Schrodinger equation for the quantum field $\vec{Y}$ by combining the Heisenberg equations of the reservoir and the field $\vec{Y}$. The equations of motion for the fields $\vec{Y}$ and $\vec{\pi}_Y$, can be obtained from the Heisenberg equations

$$\begin{aligned}\frac{\partial\vec{Y}}{\partial t} &= \frac{i}{\hbar}[H,\vec{Y}] = \frac{\vec{\pi}_Y - \alpha\vec{A} - \vec{R}}{\rho},\\ \frac{\partial\vec{\pi}_Y}{\partial t} &= \frac{i}{\hbar}[H,\vec{\pi}_Y] = -\rho\omega_0^2(\vec{r})\vec{Y} - \frac{\alpha}{\varepsilon_0}(\alpha\vec{Y})^{\parallel},\end{aligned} \tag{17}$$

so

$$\rho\frac{\partial^2\vec{Y}}{\partial t^2} + \rho\omega_0^2(\vec{r})\vec{Y} = -\alpha\frac{\partial\vec{A}}{\partial t} - \frac{\alpha}{\varepsilon_0}(\alpha\vec{Y})^{\parallel} - \frac{\partial\vec{R}}{\partial t} = \alpha\vec{E}^{\perp} + \alpha\vec{E}^{\parallel} - \frac{\partial\vec{R}}{\partial t}, \tag{18}$$

where $\vec{E}^{\perp} = -\frac{\partial\vec{A}}{\partial t}$ and $\vec{E}^{\parallel} = -\frac{(\alpha\vec{Y})^{\parallel}}{\varepsilon_0}$, are transverse and longitudinal components of the electrical field respectively. Using (14), the Heisenberg equation for $b_\nu(\vec{k},\vec{q},t)$ can be obtained as

$$\begin{aligned}\dot{b}_\nu(\vec{k},\vec{q},t) &= \frac{i}{\hbar}[H, b_\nu(\vec{k},\vec{q},t)] = -i\omega_{\vec{k}} b_\nu(\vec{k},\vec{q},t) + \\ &+ \frac{i}{\hbar\sqrt{(2\pi)^3}}\int d^3\vec{r'} f^*(\omega_{\vec{k}},\vec{r'})e^{-i\vec{q}\cdot\vec{r'}}\frac{\partial\vec{Y}(\vec{r'},t)}{\partial t}\cdot\vec{u}_\nu(\vec{q}),\end{aligned} \tag{19}$$

with the formal solution

$$\begin{aligned}b_\nu(\vec{k},\vec{q},t) &= e^{-i\omega_{\vec{k}}t} b_\nu(\vec{k},\vec{q},0) + \\ &+ \frac{i}{\hbar\sqrt{(2\pi)^3}}\int_0^t dt' e^{-i\omega_{\vec{k}}(t-t')}\int d^3\vec{r'} f^*(\omega_{\vec{k}},\vec{r'})e^{-i\vec{q}\cdot\vec{r'}}\frac{\partial\vec{Y}(\vec{r'},t')}{\partial t'}\cdot\vec{u}_\nu(\vec{q}).\end{aligned} \tag{20}$$

Substituting $b_\nu(\vec{k},\vec{q},t)$, from (20) in (18), gives

$$\rho\ddot{\vec{Y}} + \rho\omega_0^2(\vec{r})\vec{Y} + \int_0^t dt'\gamma(t-t',\vec{r})\dot{\vec{Y}}(\vec{r},t') = \alpha\vec{E}^{\perp}(\vec{r},t) + \alpha\vec{E}^{\parallel}(\vec{r},t) + \vec{\xi}(\vec{r},t), \tag{21}$$

where

$$\gamma(t-t',\vec{r})=\frac{8\pi}{\hbar c^3}\int_0^\infty d\omega\omega^3|f(\vec{r},\omega)|^2\cos\omega(t-t'), \tag{22}$$

is the memory function and describes the absorption of energy of the medium by the reservoir. The field $\vec{\xi}(\vec{r},t)$

$$\vec{\xi}(\vec{r},t)=i\sum_{\nu=1}^{3}\int d^3\vec{k}\int\frac{d^3\vec{q}}{\sqrt{(2\pi)^3}}\omega_{\vec{k}}[f(\vec{r},\omega_{\vec{k}})b_\nu(\vec{k},\vec{q},0)e^{-i\omega_{\vec{k}}t+i\vec{q}\cdot\vec{r}}-$$

$$f^*(\vec{r},\omega_{\vec{k}})b_\nu^\dagger(\vec{k},\vec{q},0)e^{+i\omega_{\vec{k}}t-i\vec{q}\cdot\vec{r}}]\vec{u}_\nu(\vec{q}). \tag{23}$$

is the noise field associated with the absorption and has a zero expectation value in the eigenstates of the reservoir. The equation (21), is called the Langevin-Schrodinger equation for the damped quantum field $\vec{Y}$. The term $\int_0^t dt'\gamma(t-t',\vec{r})\dot{\vec{Y}}(\vec{r},t')$, usually generates a damping force for $\vec{Y}$, for example if we take

$$|f(\omega)|^2=\frac{\beta c^3\hbar}{4\pi^2\omega^3}, \tag{24}$$

we obtain

$$\rho\ddot{\vec{Y}}+\rho\omega_0^2(\vec{r})\vec{Y}+\beta\dot{\vec{Y}}(\vec{r},t)=\alpha\vec{E}^{\perp}(\vec{r},t)+\alpha\vec{E}^{\parallel}(\vec{r},t)+\tilde{\vec{\xi}}(\vec{r},t), \tag{25}$$

which has a damping term proportional to the velocity and $\tilde{\vec{\xi}}(\vec{r},t)$, is the noise field (23) with the coupling function (24).

2.2 The radiation reaction

In QED, a charged particle in quantum vacuum interacts with the vacuum field and its own field known as the radiation reaction. In classical electrodynamics, there is only the radiation reaction field that acts on a charged particle in the vacuum. The vacuum and radiation reaction fields have a fluctuation-dissipation connection and both are required for the consistency of QED. For example the stability of the ground state, atomic transitions and lamb shift can only be explained by taking into account both fields. If self reaction was alone the atomic ground state would not be stable. When a quantum mechanical system interacts with the vacuum quantum field, the coupled Heisenberg equations for both the system and the quantum vacuum field give us the radiation reaction field, for example it can be shown that

the radiation reaction for a charged harmonic oscillator is $\frac{2e^2}{3c^3}$ [21]. One can find the Heisenberg equation for the annihilation operators $a_{\vec{k}\lambda}$

$$\dot{a}_{\vec{k}\lambda} = \frac{i}{\hbar}[H, a_{\vec{k}\lambda}] = -i\omega_{\vec{k}} a_{\vec{k}\lambda} + i\int d^3\vec{r'}\dot{\vec{Y}}(\vec{r'},t)\cdot\vec{e}(\vec{k},\lambda)\frac{\alpha e^{-i\vec{k}\cdot\vec{r'}}}{\sqrt{2(2\pi)^3\varepsilon_0\hbar\omega_{\vec{k}}}}, \tag{26}$$

with the formal solution

$$a_{\vec{k}\lambda}(t) = e^{-i\omega_{\vec{k}}t}a_{\vec{k}\lambda}(0) + i\int_0^t dt' e^{-i\omega_{\vec{k}}(t-t')}\int d^3\vec{r'}\dot{\vec{Y}}(\vec{r'},t')\cdot\vec{e}(\vec{k},\lambda)\frac{\alpha e^{-i\vec{k}\cdot\vec{r'}}}{\sqrt{2(2\pi)^3\varepsilon_0\hbar\omega_{\vec{k}}}}. \tag{27}$$

Substituting $a_{\vec{k}\lambda}(t)$ from (27) in $\vec{E}^\perp = -\frac{\partial\vec{A}}{\partial t}$, we obtain

$$\vec{E}^\perp = \vec{E}_0^\perp + \vec{E}_{RR}^\perp, \tag{28}$$

where

$$\vec{E}_0^\perp = i\int d^3\vec{k}\sqrt{\frac{\hbar\omega_{\vec{k}}}{2(2\pi)^3\varepsilon_0}}[a_{\vec{k}\lambda}(0)e^{-i\omega_{\vec{k}}t+i\vec{k}\cdot\vec{r}} - a^\dagger_{\vec{k}\lambda}(0)e^{+i\omega_{\vec{k}}t-i\vec{k}\cdot\vec{r}}]\vec{e}(\vec{k},\lambda), \tag{29}$$

is the transverse vacuum field and

$$\begin{aligned}\vec{E}_{RR}^\perp &= -\frac{1}{(2\pi)^3\varepsilon_0}\int d^3\vec{r'}\alpha(\vec{r'})\int d^3\vec{k}\int_0^t dt'\cos[\omega_{\vec{k}}(t-t') + \vec{k}\cdot(\vec{r}-\vec{r'})] \\ &\times \{\dot{\vec{Y}}(\vec{r'},t') - [\hat{k}.\dot{\vec{Y}}(\vec{r'},t')]\hat{k}\}\end{aligned} \tag{30}$$

is the transverse radiation reaction electrical field [21].

2.3 The Maxwell equations

The Maxwell equations can be obtained as the Heisenberg equations of $\vec{A}$ and $\vec{\pi}_F$

$$\begin{aligned}\frac{\partial\vec{A}}{\partial t} &= \frac{i}{\hbar}[H,\vec{A}] = \frac{\vec{\pi}_F}{\varepsilon_0}, \\ \frac{\partial\vec{\pi}_F}{\partial t} &= \frac{i}{\hbar}[H,\vec{\pi}_F] = (\alpha\dot{\vec{Y}})^\perp - \frac{\nabla\times\nabla\times\vec{A}}{\mu_0},\end{aligned} \tag{31}$$

after eliminating $\vec{\pi}_F$,

$$\varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = (\alpha \dot{\vec{Y}})^\perp - \frac{\nabla \times \nabla \times \vec{A}}{\mu_0}. \tag{32}$$

By defining the displacement vector field $\vec{D}$ as $\vec{D} = \vec{D}^\perp = (\alpha \vec{Y})^\perp + \varepsilon_0 \vec{E}^\perp$ and $\vec{B} = \nabla \times \vec{A}$, the equation (32) can be rewritten as

$$\nabla \times \vec{B} = \mu_0 \frac{\partial \vec{D}^\perp}{\partial t} \tag{33}$$

which is the familiar form of the Maxwell equation in a non-magnetic dielectric medium. In the absence of the external electrical charge density, we have $\vec{D}^\parallel = \varepsilon_0 \vec{E}^\parallel + (\alpha \vec{Y})^\parallel = 0$ or $\vec{E}^\parallel = -\frac{(\alpha \vec{Y})^\parallel}{\varepsilon_0}$.

3 Solution of the Heisenberg equations

The equation of motion defined in the previous sections, can be solved by using the Laplace transformation method. For any time dependent operator $g(t)$, the forward and the backward ($g^{f(b)}(t)$), Laplace transformation, are defined respectively as

$$\begin{aligned} g^f(s) &= \int_0^\infty dt g(t) e^{-st}, \\ g^b(s) &= \int_0^\infty dt g(-t) e^{-st}. \end{aligned} \tag{34}$$

Now taking the Laplace transformation of the equation (21)

$$\begin{aligned} \alpha \vec{Y}^f(\vec{r}, s) = \alpha \vec{Y}_N^f(\vec{r}, s) + \varepsilon_0 \tilde{\chi}(\vec{r}, s) \vec{E}^f(\vec{r}, s), \\ \alpha \vec{Y}^b(\vec{r}, s) = \alpha \vec{Y}_N^b(\vec{r}, s) + \varepsilon_0 \tilde{\chi}(\vec{r}, s) \vec{E}^b(\vec{r}, s), \end{aligned} \tag{35}$$

where

$$\tilde{\chi}(\vec{r}, s) = \frac{\alpha^2(\vec{r})}{\varepsilon_0 [\rho(\vec{r}) s^2 + \rho(\vec{r}) \omega_0^2(\vec{r}) + s \tilde{\gamma}(\vec{r}, s)]}, \quad \tilde{\gamma}(\vec{r}, s) = 8\pi s \int_0^\infty d\omega \frac{\omega^3 |f(\vec{r}, \omega|^2}{s^2 + \omega^2}. \tag{36}$$

The function $\tilde{\chi}(\vec{r},s)$, is the Laplace transformation of the electrical susceptibility and

$$\begin{aligned}
\alpha\vec{Y}_N^f(\vec{r},s) &= \frac{\varepsilon_0}{\alpha}\tilde{\chi}(\vec{r},s)\{\vec{\xi}^f(\vec{r},s)+\rho s\vec{Y}(\vec{r},0)+\vec{\pi}_Y(\vec{r},0)-\alpha\vec{A}(\vec{r},0)\\
&- \vec{R}(\vec{r},0)+\tilde{\gamma}(\vec{r},s)\vec{Y}(\vec{r},0)\},\\
\alpha\vec{Y}_N^b(\vec{r},s) &= \frac{\varepsilon_0}{\alpha}\tilde{\chi}(\vec{r},s)\{\vec{\xi}^b(\vec{r},s)+\rho s\vec{Y}(\vec{r},0)-\vec{\pi}_Y(\vec{r},0)+\alpha\vec{A}(\vec{r},0)\\
&+ \vec{R}(\vec{r},0)+\tilde{\gamma}(\vec{r},s)\vec{Y}(\vec{r},0)\},
\end{aligned} \tag{37}$$

are the forward and backward Laplace transformations of the noise polarization densities respectively. Another main equation can be obtained by taking the time derivative of the equation (32) and using the relation $\vec{\nabla}\times\vec{E}^{\parallel}=-\vec{\nabla}\times\frac{(\alpha\vec{Y})^{\parallel}}{\varepsilon_0}=0$

$$-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=\mu_0\alpha\frac{\partial^2\vec{Y}}{\partial t^2}+\vec{\nabla}\times\vec{\nabla}\times\vec{E}. \tag{38}$$

Equation (38), can be written in terms of the Laplace transformed components

$$\begin{aligned}
&\vec{\nabla}\times\vec{\nabla}\times\vec{E}^f(\vec{r},s)+\varepsilon_0\mu_0 s^2\tilde{\varepsilon}(\vec{r},s)\vec{E}^f(\vec{r},s)=\vec{J}^f(\vec{r},s),\\
&\vec{\nabla}\times\vec{\nabla}\times\vec{E}^b(\vec{r},s)+\varepsilon_0\mu_0 s^2\tilde{\varepsilon}(\vec{r},s)\vec{E}^b(\vec{r},s)=\vec{J}^b(\vec{r},s),
\end{aligned} \tag{39}$$

where $\tilde{\varepsilon}(\vec{r},s)=1+\tilde{\chi}(\vec{r},s)$, is the Laplace transformation of the electrical permeability and

$$\begin{aligned}
&\vec{J}^f(\vec{r},s)=\vec{\nabla}\times\vec{\nabla}\times\vec{A}(\vec{r},0)-\\
&\mu_0 s\pi_F(\vec{r},0)-\mu_0 s(\alpha\vec{Y})^{\parallel}(\vec{r},0)+\mu_0 s\alpha\vec{Y}(\vec{r},0)-\mu_0 s^2\alpha\vec{Y}_N^f(\vec{r},0),\\
&\vec{J}^b(\vec{r},s)=-\vec{\nabla}\times\vec{\nabla}\times\vec{A}(\vec{r},0)-\\
&\mu_0 s\pi_F(\vec{r},0)-\mu_0 s(\alpha\vec{Y})^{\parallel}(\vec{r},0)+\mu_0 s\alpha\vec{Y}(\vec{r},0)-\mu_0 s^2\alpha\vec{Y}_N^b(\vec{r},0),
\end{aligned} \tag{40}$$

are called the source noise densities. In reference [8], the wave equations (39) have been solved using the Green function method. In the following for simplicity, we solve the Heisenberg equations (18) and (31) for an homogeneous dielectric, that is when ρ,ω_0,α,f are independent of the position $\vec{r}$. In the case of an homogeneous and linear dielectric, in the absence of external charge density, the polarization charge density is zero and therefore the

vector field $\alpha\vec{Y}$, has only the transverse components. In this case, using the Laplace transformation of (31) and (35)

$$\vec{\nabla}\times\vec{\nabla}\times\vec{A}^f(\vec{r},s)+\mu_0\varepsilon_0 s^2\varepsilon(s)\vec{A}^f(\vec{r},s)=\mu_0 s\alpha\vec{Y}_N^f(\vec{r},s)+\mu_0\varepsilon_0 s\varepsilon(s)\vec{A}(\vec{r},0)$$
$$-\mu_0\alpha\vec{Y}(\vec{r},0)+\mu_0\vec{\pi}_F(\vec{r},0),$$
$$\vec{\nabla}\times\vec{\nabla}\times\vec{A}^b(\vec{r},s)+\mu_0\varepsilon_0 s^2\varepsilon(s)\vec{A}^b(\vec{r},s)=-\mu_0 s\alpha\vec{Y}_N^b(\vec{r},s)+\mu_0\varepsilon_0 s\varepsilon(s)\vec{A}(\vec{r},0)$$
$$+\mu_0\alpha\vec{Y}(\vec{r},0)-\mu_0\vec{\pi}_F(\vec{r},0). \tag{41}$$

The equations (41), can be solved easily using the Fourier transformation

$$\begin{aligned}
\vec{A}(\vec{r},s) &= \sum_{\lambda=1}^{2}\int d^3\vec{k}\sqrt{\frac{\hbar}{2(2\pi)^3\varepsilon_0\omega_{\vec{k}}}}a_{\vec{k}\lambda}(0)e^{i\vec{k}.\vec{r}}\frac{\mu_0\varepsilon_0(s\mp i\omega_{\vec{k}})}{\vec{k}^2+\mu_0\varepsilon_0 s^2\varepsilon(s)}\vec{e}(\vec{k},\lambda)\\
&\mp \frac{1}{\alpha}\sum_{\lambda=1}^{2}\int d^3\vec{k}\sqrt{\frac{\hbar}{2(2\pi)^3}}d_{\vec{k}\lambda}(0)e^{i\vec{k}\cdot\vec{r}}\frac{\mu_0\varepsilon_0(\rho\omega_0^2\pm is)\tilde{\chi}(s)}{\vec{k}^2+\mu_0\varepsilon_0 s^2\varepsilon(s)}\vec{e}(\vec{k},\lambda)\\
&- \frac{1}{\alpha}\int d^3\vec{k}\int\frac{d^3\vec{q}}{\sqrt{2(2\pi)^3}}\sum_{\lambda=1}^{2}f(\omega_{\vec{q}})b_\lambda(\vec{q},\vec{k},0)e^{i\vec{k}\cdot\vec{r}}\frac{\mu_0\varepsilon_0 s\tilde{\chi}(s)}{\vec{k}^2+\mu_0\varepsilon_0 s^2\varepsilon(s)}\vec{e}(\vec{k},\lambda)\\
&\pm \frac{i}{\alpha}\int d^3\vec{k}\int\frac{d^3\vec{q}}{\sqrt{2(2\pi)^3}}\sum_{\lambda=1}^{2}\frac{\omega_{\vec{q}}f(\omega_{\vec{q}})b_\lambda(\vec{q},\vec{k},0)e^{i\vec{k}\cdot\vec{r}}}{(s\pm i\omega_{\vec{q}})}\frac{\mu_0\varepsilon_0 s\tilde{\chi}(s)}{\vec{k}^2+\mu_0\varepsilon_0 s^2\varepsilon(s)}\vec{e}(\vec{k},\lambda)\\
&+ C.C,
\end{aligned}\tag{42}$$

where the upper and lower signs ($\pm$), give $\vec{A}^f(\vec{r},s)$ and $\vec{A}^b(\vec{r},s)$, respectively. Now $\vec{A}(\vec{r},t)$ for $t>0$ is the inverse Laplace transformation of $\vec{A}^f(\vec{r},s)$ and $\vec{A}(\vec{r},-t)$ for $t>0$, is the inverse Laplace transformation of $\vec{A}^b(\vec{r},s)$, that is

$$\begin{aligned}
\vec{A}(\vec{r},t) &= SOR[e^{st}\vec{A}^f(\vec{r},s)], \qquad t>0,\\
\vec{A}(\vec{r},t) &= SOR[e^{-st}\vec{A}^b(\vec{r},s)], \qquad t<0,
\end{aligned}\tag{43}$$

where $SOR[f(s)]$, means the sum of the residues of a complex function f. If the residues of the complex functions $\frac{s\mp i\omega_{\vec{k}}}{\vec{k}^2+\mu_0\varepsilon_0 s^2\varepsilon(s)}$, $\frac{(\rho\omega_0^2\pm is)\tilde{\chi}(s)}{\vec{k}^2+\mu_0\varepsilon_0 s^2\varepsilon(s)}$ and $\frac{s\tilde{\chi}(s)}{\vec{k}^2+\mu_0\varepsilon_0 s^2\varepsilon(s)}$, with respect to s have negative real parts, as usually it is the case for an absorptive dielectric, then we find the following behaviour in $t\to\infty$

$$\begin{aligned}
\vec{A}(\vec{r},t) &= \frac{\mu_0\varepsilon_0}{\alpha}\int d^3\vec{k}\int\frac{d^3\vec{q}}{\sqrt{(2\pi)^3}}\sum_{\lambda=1}^{2}\frac{\omega_{\vec{q}}^2 f(\omega_{\vec{q}})b_\lambda(\vec{q},\vec{k},0)\tilde{\chi}(-i\omega_{\vec{q}})}{\vec{k}^2-\mu_0\varepsilon_0\omega_{\vec{q}}^2\varepsilon(-i\omega_{\vec{q}})}e^{-i\omega_{\vec{q}}t+i\vec{k}\cdot\vec{r}}\vec{e}(\vec{k},\lambda)\\
&+ C.C.
\end{aligned}\tag{44}$$

We can find the polarization field $\vec{Y}(\vec{r},t)$ by substituting $\vec{A}^f(\vec{r},s)$ and $\vec{A}^b(\vec{r},s)$ from (42) in (35) and then taking the inverse Laplace transformation of $\vec{Y}^f(\vec{r},s)$ and $\vec{Y}^b(\vec{r},s)$.

4 Conclusion

By taking a reservoir with continuous degrees of freedom and a suitable Hamiltonian, we could give a consistent quantization scheme for electromagnetic field in an absorptive dielectric medium, in the frame work of the damped polarization model. In this model, the damped polarization field, was satisfying the quantum Langevin equation containing the electrical field as the source term. Using the Laplace transformation, the quantum Langevin equation solved and the susceptibility of the dielectric obtained in terms of the coupling function between the reservoir and the polarization field. Equations containing a noise term for the vector potential and the electrical field obtained and solved which led to explicit forms of the vector potential and the electrical field.

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